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THE EFFECT OF A TRANS-PLUTONIAN PLANET ON HALLEY'S COMET

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An orbit and mass for a hypothetical trans-Plutonian planet is determined which reduces the residuals in the time of perihelion passage of Halley's comet at the seven apparitions from 1910 to 1456 by 93%. The effect of this hypothetical planet on the major planets is briefly discussed, and it is shown that the residuals of two similar periodic comets, Olbers and Pons-Brooks, are also improved.

Key words: comet — Halley's comet — planet — trans-Plutonian

I. Introduction

There is no logical reason to suppose Pluto to be the outermost planet of the solar system. Indeed, there is a great deal of evidence indicating that it is not, and the literature of the last 100 years records numerous attempts to predict planets beyond Neptune and Pluto, some of which have led to telescopic searches. Among the early predictions were Forbes (1880), Todd (1880), and Gaillot (1909); and more recently, Sevin (1946, 1949), Strubell (1953), and Kritzinger (1958, 1963). But the most famous were Pickering's (1909, 1928) with its discouraging conclusion and, of course, the fruitful prediction of Percival Lowell (1915). Throughout this literature, the most consistent message is that there must be two trans-Neptunian planets. One of these is now Pluto and the other one appears to be about twice the distance of Neptune from the sun.

Whether Pluto was actually Lowell's planet X, or its discovery an extraordinary coincidence as Brown (1930, 1931) claimed, or fortuitous as Kourganoff (1941) indicates in his refutation of Brown, is still an intriguing question. But even more intriguing is the consistent failure of the theories of Neptune's motion to represent the observations. As indicated by Rawlins (1970), the unexplained residual in the orbit of Neptune may be due to an alien perturbation; but years may have to pass before planetary residuals will reveal the position of a trans-Plutonian planet.

However, Halley's comet, which has been observed for over 2000 years, might provide the necessary record from which the orbit of a planet beyond Pluto can be found. Obviously this is not a continuous record. Most are perihelion passages at approximately 76-year intervals and all but the last four are pretelescopic. Nevertheless, there are two millennia of recorded observations. Furthermore, the effectiveness of a disturbing body is a direct function of the eccentricity of the disturbed body, and P/Halley, with its large eccentricity, would be more highly disturbed by a trans-Plutonian mass than the major planets.

In order to link several apparitions of Halley's comet, Brady and Carpenter (1971) found it necessary to include a secular term in the equations of motion. This secular term needs no apology. Its purpose was to provide a good prediction for the 1986 return — but it does much more. It represents the motion of the comet for a very long period of time, possibly for more than 2000 years, and in many cases better than indicated by the presently accepted catalog values. But even if the secular term needs no defense it would be more meaningful if it had a physical interpretation. To attribute it to nongravitational forces is one possibility and due mainly to the work of Marsden (1968), it is now generally accepted that there are such forces affecting many short-period comets. With comets of longer periods, however, another possible explanation is an alien perturbation, suggesting that the force is gravitational after all.

What follows is a report on a numerical experiment to determine if it is possible to represent the motion of Halley's comet over a long period of time by including the effect of a hypothetical

planet beyond Pluto. The solution is a numerical-graphical one, and this brief summary greatly oversimplifies the process, failing completely to give a true picture of the actual amount of computing.

The first step determined an approximation for the semimajor axis and mass of the hypothetical planet from the residuals of Halley's comet. With this semimajor axis and mass, and with the eccentricity and inclination equal to zero, the second step determined a starting longitude. With this semimajor axis, mass, and starting longitude, and with the eccentricity still zero, the third step determined the inclination and the longitude of the node. With these five elements known, the last step determined the eccentricity and the argument of perihelion.

The integrations were all done on a CDC 6600 computer with an n-body code (McMahon 1970), using the planetary starting values of Lieske (1967) with the masses of the planets modified according to Clemence (1965).

II. Semimajor Axis and Mass

Two orbits of Halley's comet were defined by Brady and Carpenter (1971). The first, designated Orbit 12a, was purely gravitational and included the effect of the nine planets. The second, designated Orbit 40B + ϵ_6, included the nine planets and a secular term which had been added as a multiplier to the solar term in the equations of motion. With Orbit 12a it was not possible to represent the motion of Halley's comet for more than two apparitions, the third apparition being missed by four days. While the inevitable difference between observation and calculation can seldom be traced to its source with absolute conviction, in this case the four-day error was so large and stable as to point with assurance to the inadequacy of the mathematical model. It was surely not a failure of the law of gravity and therefore had to be a missing term in the equations of motion. The added secular term of Orbit 40B + ϵ_6 does not in itself have a reasonable physical interpretation but nevertheless accounts in some way for this missing term, since it represents the motion of the comet for a long period of time.

The problem then was to find a planetary orbit and mass beyond Pluto to include in the perturbations of Halley's comet which would do as well as the secular term. Yet the effect of this planetary orbit on the nine planets must be such that small changes in their starting values could restore the present accuracy of their orbits.

Table I compares a set of adopted perihelion passages for Halley's comet with those derived from the two mentioned orbits. Column 1 gives the date of the apparition. Column 2 gives the adopted Julian Date of the perihelion passage. Column 3 gives the error in days between the value in column 2 and the time of perihelion obtained from a continuous integration using Orbit 12a. Column 4 gives the error in days between the value in column 2 and the time of perihelion obtained from a continuous integration using Orbit 40B + ϵ_6. Column 5, to first order, shows the effect of Pluto on Halley's comet by giving the difference between two integrations of Orbit 12a, one of which does not include Pluto. Column 6 gives the difference between an integration with Orbit 40B + ϵ_6 and Orbit 12a. To first order then, column 6 shows the effect of the secular term on Halley's comet.

The bottom curve of Figure 1 is a plot of the data from column 6, Table I. The top curve of Figure 1 is a plot of the data from column 5, Table I.

If the term missing from the mathematical model is due to a planet beyond Pluto, the bottom curve of Figure 1 must contain information about this planet. The top curve of Figure 1 is due entirely to Pluto and must contain similar information about Pluto.

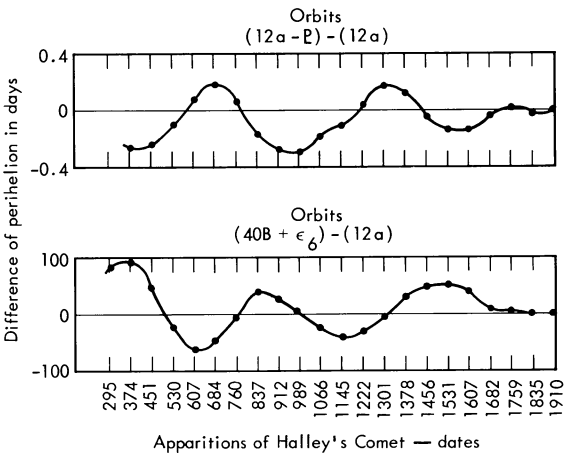


FIG. 1 — Effect of Pluto and the hypothetical planet on Halley's comet.

TABLE I

PERIHELION PASSAGES AND RESIDUALS OF HALLEY'S COMET

Year of apparition [1]		T Julian Date (adopted) [2]	ΔT Orbit 12a [3]	ΔT Orbit 40B + ϵ_6* [4]	ΔT Orbits (12a − P) − (12a) [5]	ΔT Orbits (40B + ϵ_6) − (12a) [6]
1910	241	8781.67872	$0^d.0$	$0^d.0$	$0^d.0000$	$0^d.00$
1835	239	1598.93817	0.0	0.0	-0.0088	0.00
1759	236	3592.55178	+4.2	0.0	+0.0180	+4.17
1682	233	5655.79506	+8.2	0.0	-0.0358	+8.21
1607	230	8303.71595	+42.6	-0.1	-0.1312	+42.66
1531	228	0492.792	+53.5	+0.2	-0.1214	+53.34
1456	225	3021.202	+49.6	-0.8	-0.0482	+50.35
1378	222	4684.770	+29.4	-2.1	+0.1315	+31.55
1301	219	6543.700	-8.7	-3.7	+0.1869	-5.03
1222	216	7646.0	-51.6	-19.1	+0.0470	-32.48
1145	213	9378.	-40.5	+1.1	-0.0982	-41.65
1066	211	0500.	-12.8	+7.5	-0.1791	-20.29
989	208	2535.	+0.8	-0.5	-0.283	+1.27
912	205	4366.	+28.7	+2.4	-0.270	+26.33
837	202	6828.	+37.8	-2.9	-0.164	+40.67
760	199	8810.	+14.8	+20.2	+0.066	-5.38
684	197	1199.	-18.1	+30.3	+0.193	-48.34
607	194	2843.	-60.9	+1.8	+0.081	-62.68
530	191	4959.	+27.7	+49.1	-0.083	-21.44
451	188	5969.	+55.2	+7.7	-0.240	+47.54
374	185	7705.	+95.2	+0.4	-0.263	+94.73
295	182	8903.	+69.3	-15.5		+84.83

* $\epsilon_6 = 2.635 \times 10^{-9}$.

A Fourier analysis of the bottom curve of Figure 1, with the interval from 1835 back to 295 for the basic period of 2π, gives a period of 513 years for the dominant harmonic. This corresponds to a mean distance of 65 a.u. If the masses of Pluto and the hypothetical planet are assumed proportional to the amplitudes of the upper and lower curves respectively, and inversely proportional to the square of their mean distances, the mass of the hypothetical planet is equal to 0.0003 solar masses.

This mean distance and mass is used only as a first approximation in what follows. It is realized that the harmonic analysis of a curve such as this is difficult and, once a basic period is picked, the cards are stacked against certain other periods. But this is the way the process developed. These quantities, especially the mass, changed considerably during the process to be described and the steps that follow depend very little on the harmonic analysis.

III. Starting Longitude

The procedure in this step was to assume the hypothetical planet to be in a circular orbit in the plane of the ecliptic and see if a starting longitude could be found at an initial epoch in 1911 such that an 11-body, purely gravitational integration would give small residuals for the time of perihelion passage of Halley's comet at all apparitions back to 1456. The choice of the interval from 1910 to 1456 was based on two considerations; first, it was roughly equal to the period of the hypothetical planet, and second, it was a short enough interval that the several hundred integrations could be done with a reasonable amount of computer time.

The hypothetical planet was given starting longitudes at various points around the ecliptic and the results plotted for each apparition. The residual ΔT, at each apparition, was plotted against the starting longitude at the epoch. This was done for three different masses, 0.0003, 0.0005, 0.0009, various distances in the neighborhood of 65 a.u., and for several different orbits of Halley's comet. The results for three of these comet orbits are shown in Figures 2 and 3 for a mass of 0.0009, and a distance of 65.491 a.u. It can be seen from these figures that with a longitude of 150°.75 for the starting value of the hypothetical planet in 1911, the residuals of

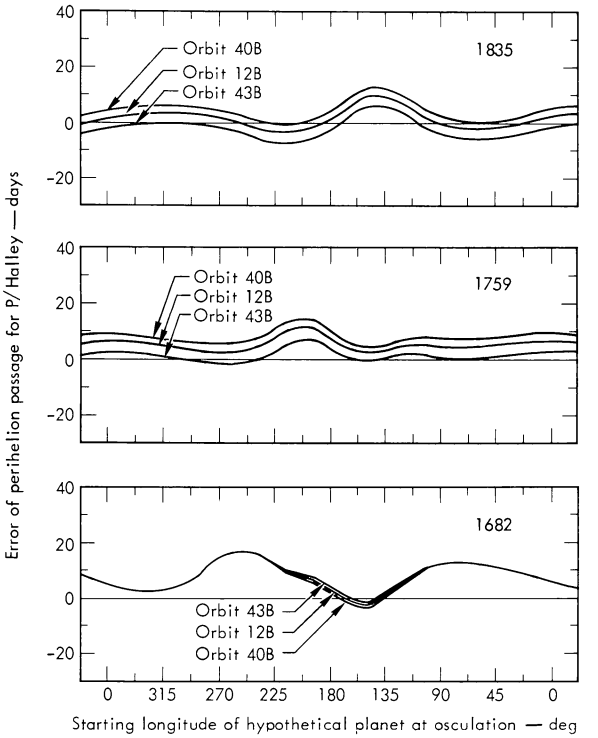


FIG. 2 — Error of perihelion for P/Halley vs. starting longitude of hypothetical planet for the apparitions of 1835, 1759, and 1682.

Halley's comet come close to zero back to the 1456 apparition with the exception of the 1835 apparition. The plot for 1910 is not shown because the ΔT residuals are always close to zero.

Thus, even for the restricted case of a circular orbit in the plane of the ecliptic there is an initial longitude for the hypothetical planet at the 1911 epoch which greatly reduces the residuals back to 1456 in the time of perihelion, although a very large mass is required.

IV. Inclination and Longitude of the Node

With an eccentricity of zero, a semimajor axis of 65.491 a.u. and a mass of 0.0009, the effect of inclination was examined next. Since the inclination also involves the longitude of the node, the problem becomes three dimensional. The error of perihelion passage, ΔT, for P/Halley was to be a function of the hypothetical planet's i and Ω. From the previous experiment it looked as if an initial longitude of 150°.75 was the correct region for the starting position of the hypothetical planet. Since a low inclination was anticipated, it was decided for each value of Ω to

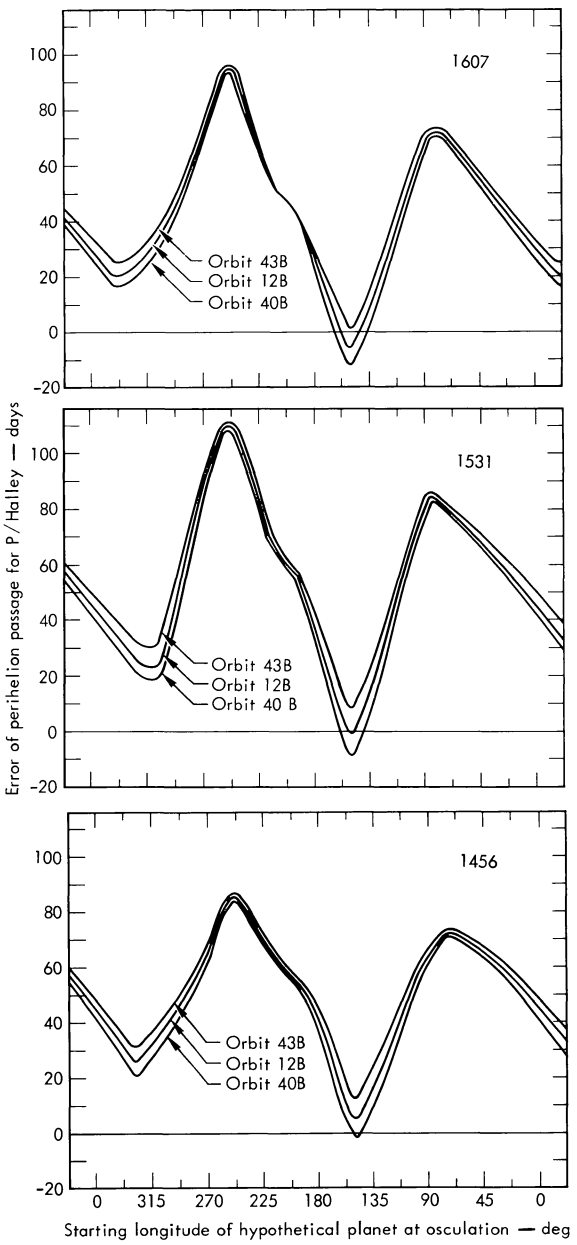


FIG. 3 — Error of perihelion for P/Halley vs. starting longitude of hypothetical planet for the apparitions of 1607, 1531, and 1456.

make $\omega = 150°.75 - \Omega$ where ω is the argument of perihelion, and arbitrarily assign $T = t_0 =$ the epoch. This would start the hypothetical planet close to longitude 150°.75, i.e., above or below the point of the ecliptic equal to 150°.75. The result was a three dimensional plot with i on the abscissa, Ω on the ordinate, and the value of ΔT recorded in the i, Ω space, with one plot for each apparition. The desired i and Ω would be the values which gave zero solutions common to all apparitions.

This was done with four different orbits of Halley's comet, in order that the best orbit of the comet could be interpolated from them. Figure 4 shows these plots for one of the comet orbits, here designated 52B. The solid lines indicate the zero solutions and some of the nonzero ΔT values are shown.

At first only inclinations from 0° to 20° were looked at, but no zero surface could be found for 1759. All other apparitions had zero solutions but no common solution. As can be seen from Figure 4b, there is no solution for 1759 until almost 70° of inclination. As the inclination got larger the starting position of the hypothetical planet had less and less connection with the starting longitude discussed in the previous section. When the inclination became retrograde the algorithm for computing starting values was maintained, although this moved the starting point away from 150°.75. The problem had now become one of finding zero solutions in the i, Ω plane, regardless of previous experiments in the plane of the ecliptic.

By an inspection of Figure 4 it can be seen that there are two common zero solutions. Both are near $i = 120°$. The first solution is at $\Omega = 120°.75$ and the second solution is almost 180° away at $\Omega = 290°.75$. Comparing these two orbits against Orbit 12a, we see that the first solution reduces the residuals in the time of perihelion by 86% and the second solution reduces the residuals by 85%.

V. Eccentricity and Argument of Perihelion

In this experiment very little progress has been made in examining the effect of eccentricity. The plan is to hold the other elements constant, and plot ΔT as a function of e and ω. This will be a three-dimensional plot similar to the i, Ω plots of the previous section but with e on the abscissa and ω on the ordinate. With e not equal to zero, the radius vector, r_0, which was constant when e was equal to zero, will be held fixed at the epoch and therefore the semimajor axis will vary depending on e and ω. Plots will be made for various r_0 and for both solutions. In a preliminary experiment with the first solution, some

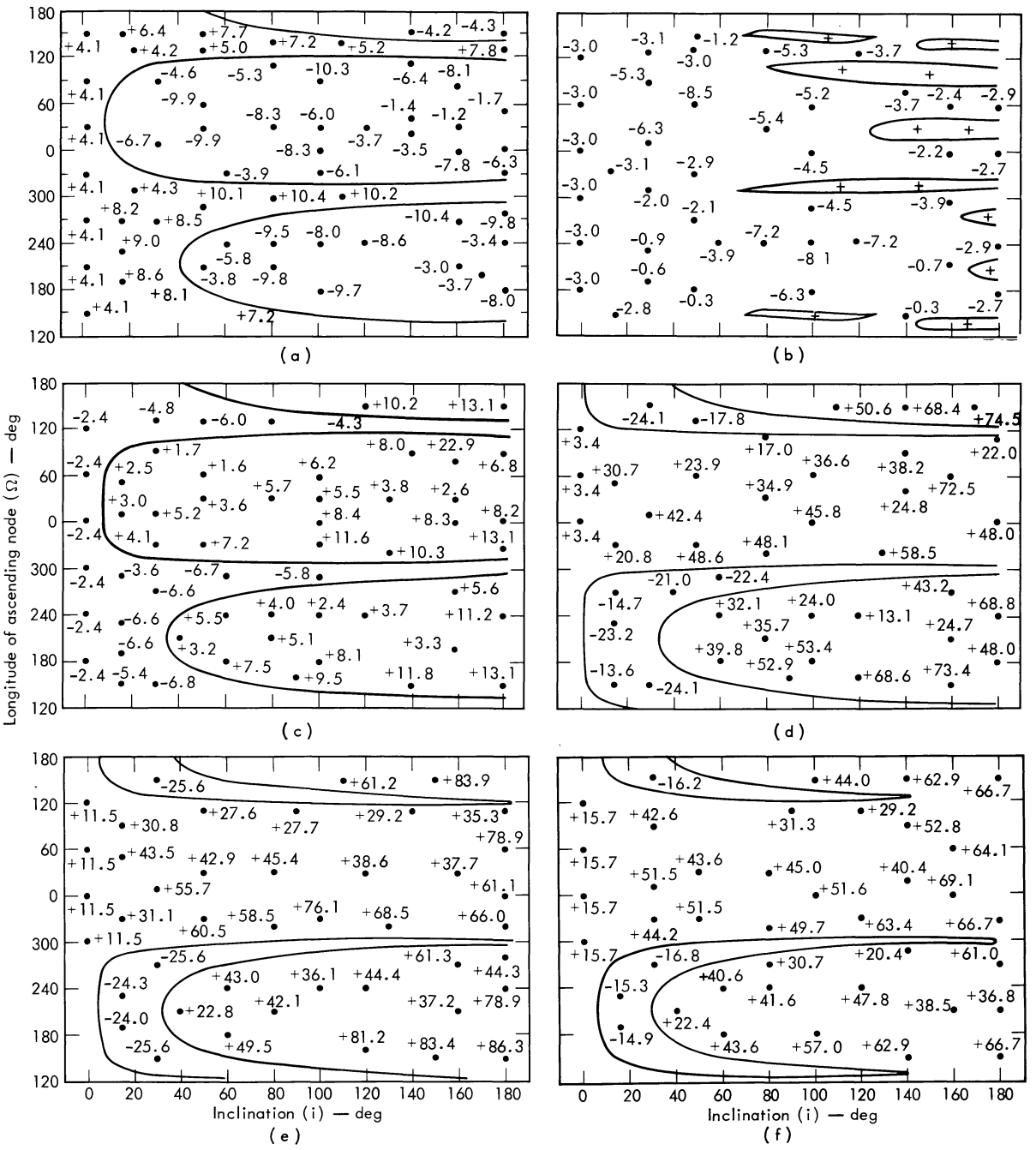


FIG. 4 — Error of perihelion for P/Halley as a function of inclination and longitude of the node. $e = 0$, $a = 65.491$, and mass = 0.0009; apparitions of (a) 1835, (b) 1759, (c) 1682, (d) 1607, (e) 1531, (f) 1456.

improvement in the residuals occurs for $e = 0.07$, $\omega = 181°$, $\Omega = 115°.75$ and $r_0 = 63.491$; but the effect on the position of the hypothetical planet in the years near the epoch is negligible. It is this experimental orbit which is discussed in the following sections.

VI. Orbit of the Hypothetical Trans-Plutonian Planet

The orbit given below for the hypothetical planet reduces the residuals in the time of perihelion for Halley's comet by 93% of Orbit 12a (1910 to 1456).

Epoch of osculation = 2419326.5
= 1911 October 16.5 = t_0

Equator and Equinox of 1950.0

$x_0 = -\ 8.91701\ 41273\ 020$
$y_0 = +40.22014\ 79257\ 339$
$z_0 = +48.31080\ 25278\ 550$
$\dot{x}_0 = +\ 0.00128\ 74352\ 0573$
$\dot{y}_0 = -\ 0.00120\ 10180\ 1805$
$\dot{z}_0 = +\ 0.00113\ 50090\ 8700$

$e = 0.07$ $\omega = 181°$
$a = 59.93575$ $\Omega = 115\overset{\circ}{.}75$
$r_0 = 63.491$ $i = 120°$

$T = 1635$
Mass = 0.0009
Period = 464 years

The starting values for the Halley's comet orbit designated Orbit 52B which produced the residuals shown in Figure 4 are as follows.

Epoch of osculation = 2419326.5
= 1911 October 16.5 = t_0

Equator and Equinox of 1950.0

$x_0 = -5.75587\ 07435\ 155$
$y_0 = +2.86180\ 52515\ 000$
$z_0 = -0.88527\ 66122\ 360$
$\dot{x}_0 = -0.00614\ 88611\ 3277$
$\dot{y}_0 = +0.00606\ 94635\ 8546$
$\dot{z}_0 = -0.00020\ 74459\ 2931$

Table II shows the effect of this hypothetical planet on Halley's comet. Column 1 shows the year of the apparition. Column 2 shows the error in days from Orbit 12a using the nine planets only. Column 3 shows the error in days from Orbit 52B using the nine planets and the hypothetical planet, the orbit of which is shown above.

The residuals of Halley's comet are fairly sensitive to small changes in the orbit of the hypothetical planet but the position of the planet in the neighborhood of osculation is altered very little relative to the earth and thus the range of solution is small.

A predicted time of perihelion cannot have the authority of an observed apparition, no matter how convincingly accurate it may be. For this reason very little attention has been paid here to the 1986 apparition which, however, could provide a way to choose between the two solutions for the hypothetical planet. A plot, like those of Figure 4, is being made for the 1986 apparition in order to find where the zero solutions are. The time of perihelion passage for this plot is assumed to be 1986 February 9.39474 (Brady and Carpenter 1971).

TABLE II

RESIDUALS OF HALLEY'S COMET WITH AND WITHOUT THE TRANS-PLUTONIAN PLANET

Year of apparition	ΔT Orbit 12a 9 planets	ΔT Orbit 52B 10 planets
1910	0$\overset{d}{.}$0	0$\overset{d}{.}$0
1835	0 .0	−0 .8
1759	+ 4 .2	−0 .8
1682	+ 8 .2	−3 .0
1607	+42 .6	−2 .7
1531	+53 .5	+2 .5
1456	+49$\overset{d}{.}$6	+3$\overset{d}{.}$9

VII. The Effect of the Hypothetical Planet on the Other Planets

If a trans-Plutonian planet is to be plausible, its effect on the other planets must be small enough that new starting values can restore their orbits to at least the present state of accuracy. A program to demonstrate this by adjusting the starting values to the observations, a lengthy undertaking, is now in progress. Adjusting the starting values to the present theories would be much easier, but not entirely satisfactory, because the theories themselves have systematic errors and the results might not be conclusive. Furthermore, the orbit and mass for the hypothetical planet given here may change as the argument of perihelion and the eccentricity is further investigated and as the orbit is extended backward and forward in time.

However, the amount by which this hypothetical planet alters the orbits of the other planets can be shown by comparing two integrations: one, a nine-body integration with planets Mercury through Pluto; the other, a ten-body integration which includes the hypothetical planet. Table III, showing the residuals in right ascension and declination in 1835 after an integration of 76 years, demonstrates the effect, as seen from the earth, of this hypothetical planet on each of the other planets; i.e., the approximate amount that must be removed by new starting values.

TABLE III

EFFECT OF TRANS-PLUTONIAN PLANET ON THE MAJOR PLANETS
(Amount that must be removed by new starting values)

Planet	After 76 yr	
	$\Delta\alpha \cos \delta$	$\Delta\delta$
Mercury	0″	0″
Venus	0	0
Mars	+ 1	0
Jupiter	− 2	0
Saturn	−11	+ 5
Uranus	− 4	− 6
Neptune	+40	−12
Pluto	+47	−33

TABLE IV

RESIDUALS OF P/OLBERS AND P/PONS-BROOKS WITH AND WITHOUT THE TRANS-PLUTONIAN PLANET

Year of apparition	P/Olbers ΔT Orbit A 9 planets	P/Olbers ΔT Orbit B 10 planets
1815	0^{d}0	0^{d}0
1887	−0 .5	0 .0!!
1956	+5 .0	+1 .5
	P/Pons-Brooks	
	Orbit C	Orbit D
1954	0 .0	0 .0
1884	−4 .6	0 .0
1812	−7^{d}2	−4^{d}1

Two versions of the n-body code were used. One was a normal n-body integration in which every planet perturbed every other planet. The second version was a modification in which all ten planets (Mercury through Pluto plus the hypothetical planet) perturbed the comet, the nine planets (Mercury through Pluto) perturbed the hypothetical planet, and the hypothetical planet did not perturb the nine planets. Because the starting values of the nine planets are adjusted to represent observations, the perturbations on the comet and the hypothetical planet are more realistic with this version. However, the residuals in ΔT for Halley's comet, back to 1456, are the same with either version.

Not all of the residual for Neptune and Pluto need be removed because the starting values for these planets are not well determined. Neptune has only completed three-quarters of a revolution covered by observations, while the observations of Pluto cover less than one-quarter of its orbit.

VIII. The Effect of the Hypothetical Planet on P/Olbers and P/Pons-Brooks

Starting values can always be found for a comet which will fit two adjacent perihelion passages. If the effect of a hypothetical planet on a comet is to be noticeable, the comet must have been observed at three or more apparitions and it must have an aphelion distance great enough that the effect can be detected above the observational errors.

There are only two comets, besides Halley's comet, that meet these conditions. They are Comet Olbers and Comet Pons-Brooks. Each has been observed at three apparitions and both have aphelion distances and eccentricities similar to Halley's comet, but here the resemblance ceases. They move in direct orbits, while P/Halley is retrograde and they are at perihelion when P/Halley is near aphelion. Yet they must feel the effect of a trans-Plutonian planet just as P/Halley does.

The orbits of these two comets were integrated in the same manner as Halley's comet, with and without the hypothetical planet. The effect of the hypothetical planet is shown in Table IV. The columns are self-explanatory. Orbit A is from Rasmusen (1948) with osculating epoch in 1815. Orbit C is from Musen (1954) with osculating epoch in 1953. Orbits B and D are similar to Orbits A and C, respectively, with the exception of small adjustments in their starting values to reduce the residual at the second perihelion. Otherwise, no attempt has been made to improve the orbits.

IX. The Lowell Observatory Trans-Neptunian Planet Search

Following the discovery of Pluto in 1930 at the Lowell Observatory, Tombaugh (1961) continued the trans-Neptunian planet search for 16 years, photographing and blinking most of the sky down to the 16th or 17th magnitude. If the hypothetical planet discussed here had the albedo

of Pluto and any reasonable density it would be 13th or 14th magnitude. Why then could it have been overlooked by the Lowell Observatory search?

Figure 5 gives a possible answer. It shows the area of the sky covered by Tombaugh's search. The track of the hypothetical planet is superimposed as the dark line between declination +60° and +67°. The lower end was its position in 1929, the upper end was its position in 1946. As can be seen, its entire path during the 16-year search was outside the area covered.

X. Summary

This experiment indicates that a planet at twice the distance of Neptune and a mass three times that of Saturn, inclined 120° to the ecliptic, will keep the motion of Halley's comet close to reality for over 450 years. In addition, it improves the motion of two other similar comets.

Clearly, this hypothetical planet can have no observable effect on the very short-period comets such as Encke and d'Arrest which, according to Marsden (1968), show strong nongravitational forces. In fact, some of the remaining residual in Halley's comet might be due to this cause.

The residuals of Halley's comet over the interval considered are reduced by 93% but those that remain are systematic and resemble the bottom curve of Figure 1 with a greatly reduced amplitude. Possibly a better value of the eccentricity can further reduce these remaining residuals, or they could be an indication of yet another unknown planet.

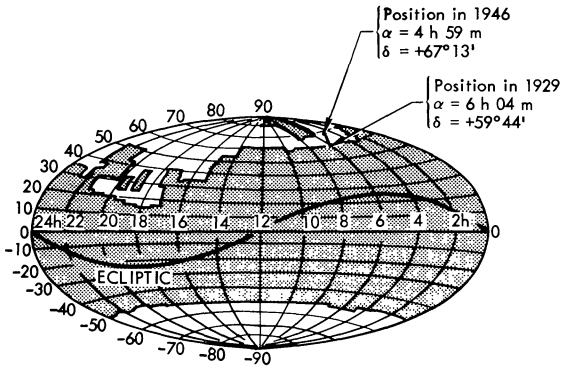


FIG. 5 — Area covered by trans-Neptunian planet search made at the Lowell Observatory, 1929–45 (shown stippled). The positions of the hypothetical trans-Plutonian planet are shown for 1929 and 1946.

In conclusion I should like to express to Sidney Fernbach my gratitude for his support and my thanks for his advice and criticism during the preparation of the manuscript. My sincere thanks are also due Frank McMahon for his *n*-body code and much help which was freely given. I am indebted to my wife for translations of many of the references. This work was done under the auspices of the U.S. Atomic Energy Commission.

REFERENCES

Brady, J. L., and Carpenter, E. 1971, *A.J.* **76**, 728.
Brown, E. W. 1930, *Proc. Nat. Acad. Sci.* **16**, 364.
—— 1931, *M.N.R.A.S.* **92**, 80.
Clemence, G. 1965, *Annual Rev. of Astr. and Astrophysics* 3, 101.
Forbes, G. 1880, *Proc. Roy. Soc. Edinburgh* **10**, 426; **11**, 89.
Gaillot, A. 1909, *Comptes Rendus* **148**, 754.
Kourganoff, V. 1941, *Bull. Astron. Tome XII - Fasc. IV*, 147.
Kritzinger, H. H. 1958, *Nbl A.* Z **13**, 3 (No. 1).
—— 1963, *Die Sterne* **39**, 6 (No. 1-2).
Lieske, J. 1967, *Jet Propulsion Lab. Tech. Report* 32-1206.
Lowell, P. 1915, *Mem. Lowell Obs.* **1** (No. 1).
Marsden, B. G. 1968, *A.J.* 73, 367.
McMahon, F. H. 1970 (private communication).
Musen, P. 1954, *Harvard Obs. Announcement Card* (No. 1249).
Pickering, W. H. 1909, *Harvard College Obs. Annals LXI*, ii, 113.
—- 1928, *Popular Astronomy* **36**, 417 (No. 7).
Rasmusen, H. Q. 1948, *Pub. Kobenhavn's Obs.* **147**.
Rawlins, D. 1970, *A.J.* **75**, 856.
Sevin, E. 1946, *Comptes Rendus* **223**, 469; 653.
—— 1949, *Bull. de l'association des anciens eleves de l'ecole Polytechnique*, No. 19, 3.
Strubell, W. 1953, *Die Sterne* **29**, 111 (No. 5-6).
Todd, D. P. 1880, *American J. Sci.* **120**, 231.
Tombaugh, C. W. 1961, in *Planets and Satellites*, G. P. Kuiper and B. M. Middlehurst, eds. (Chicago: University of Chicago Press), chap. 2.